Game Theory Intro

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Defining Games - Key Ingredients

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  - People? Governments? Companies? Somebody employed by a Company?...

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  - Enter a bid in an auction? Decide whether to end a strike? Decide when to sell a stock? Decide how to vote?
- Payoffs: what motivates players?
  - Do they care about some profit? Do they care about other players?...

Defining Games - Two Standard Representations

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- Extensive Form Includes timing of moves (later in course)
  - Players move sequentially, represented as a tree
    - Chess: white player moves, then black player can see white's move and react…
  - Keeps track of what each player knows when he or she makes each decision
    - Poker: bet sequentially – what can a given player see when they bet?

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  - Utility function or Payoff function for player i: $u_i : A \mapsto \mathbb{R}$
    - $u = (u_1, \ldots, u_n)$, is a profile of utility functions

Normal Form Games - The Standard Matrix Representation

- Writing a 2-player game as a matrix:
  - "row" player is player 1, "column" player is player 2
  - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
  - cells listing utility or payoff values for each player: the row player first, then the column

Games in Matrix Form

Here's the TCP Backoff Game written as a matrix

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

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- Utility function for player i:
  - $u_i(a) = 1$ if $\#\{j : a_j = Revolt\} \geq 2,000,000$
  - $u_i(a) = -1$ if $\#\{j : a_j = Revolt\} < 2,000,000$ and $a_i = Revolt$
  - $u_i(a) = 0$ if $\#\{j : a_j = Revolt\} < 2,000,000$ and $a_i = Not$

Summary: Defining Games

- For Now: Normal Form (Strategic Form, Matrix Representation...)
  - Players: N
  - Actions: A_i
  - Payoffs: u_i
- Later: Extensive Form
  - Timing: in what order do things happen?
  - Information: what do players know when they act